Advanced Automatic Control

“If you have a smart project, you can say "I'm an engineer"”

Lecture 3

Staff boarder

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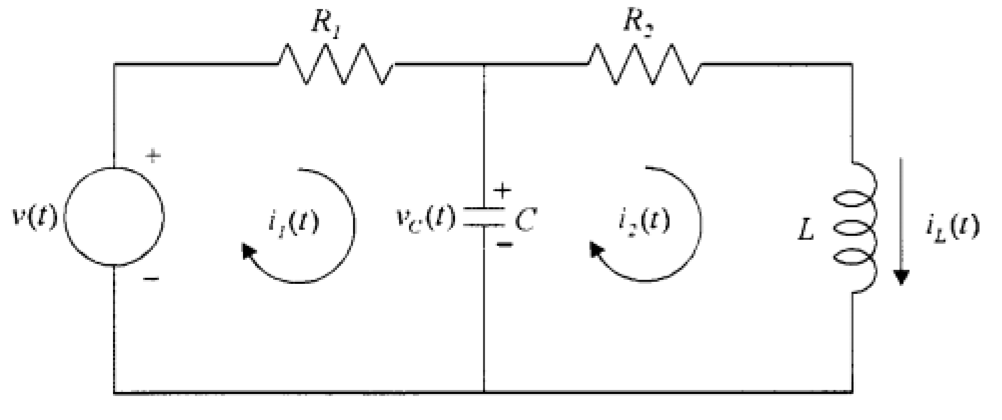
Advanced Automatic Control
MDP 444

- **Lecture aims:**
  - Facilitate combining and manipulating differential equations
  - Identify the equations of motion of systems
  - Understand the mathematical modeling of all systems and combination

Modeling of electrical system

- Kirchhoff's laws:
  1. Kirchhoff's voltage law. The algebraic sum of the voltages in a loop is equal to zero.
  2. Kirchhoff's current law. The algebraic sum of the currents in a node is equal to zero.



Modeling of electrical system

- Kirchhoff's laws:

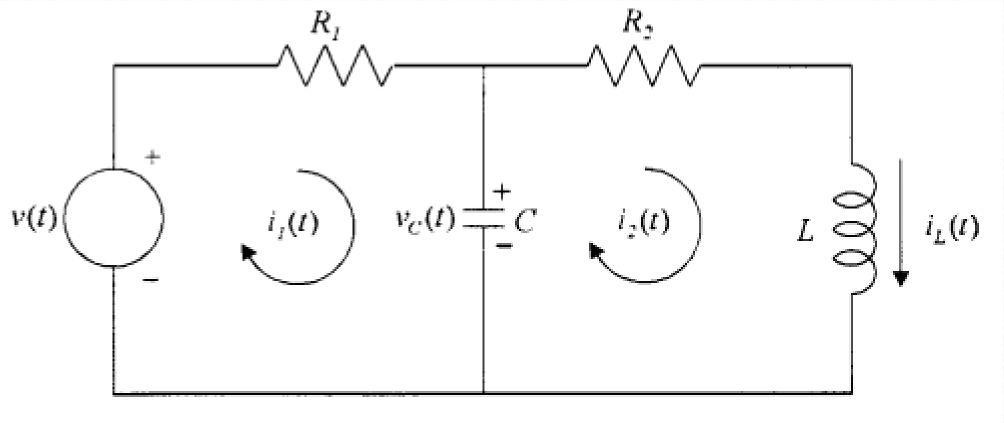
  Kirchhoff's voltage law. The algebraic sum of the voltages in a loop is equal to zero.

- Loop (1)
- $V(t) = V_R + Vc$

$$R_1 i_1(t) + \frac{1}{C}\int_0^t i_1(t)\,\mathrm{d}t - \frac{1}{C}\int_0^t i_2(t)\,\mathrm{d}t = v(t)$$

- Loop (2)
- $0 = V_R + Vc$

$$-\frac{1}{C}\int_0^t i_1(t)\,\mathrm{d}t + R_2 i_2(t) + L\frac{\mathrm{d}i_2}{\mathrm{d}t} + \frac{1}{C}\int_0^t i_2(t)\,\mathrm{d}t = 0$$



Modeling of electrical system

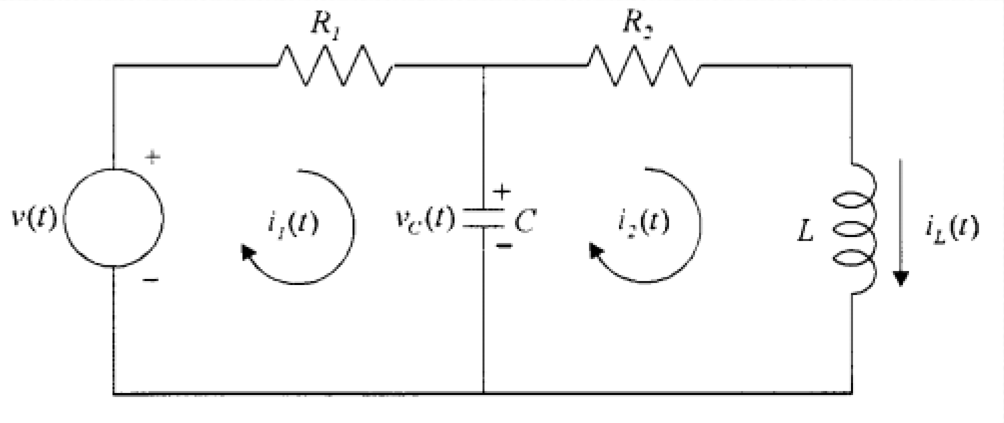
- Transfer from time domain to frequency domain:

$$R_1 i_1(t) + \frac{1}{C}\int_0^t i_1(t)\,\mathrm{d}t - \frac{1}{C}\int_0^t i_2(t)\,\mathrm{d}t = v(t)$$

$$\left[R_1 + \frac{1}{Cs}\right] I_1(s) - \frac{1}{Cs} I_2(s) = V(s)$$

$$-\frac{1}{C}\int_0^t i_1(t)\,\mathrm{d}t + R_2 i_2(t) + L\frac{\mathrm{d}i_2}{\mathrm{d}t} + \frac{1}{C}\int_0^t i_2(t)\,\mathrm{d}t = 0$$

$$-\frac{1}{Cs} I_1(s) + \left[R_2 + Ls + \frac{1}{Cs}\right] \mathrm{I}_2(s) = 0$$



- Transfer function

$$\frac{I_2(s)}{V(s)} = \frac{Cs}{(R_1Cs+1)(LCs^2+R_2Cs+1)-1} = \frac{1}{R_1LCs^2 + (R_1R_2C+L)s + R_1 + R_2}$$

Modeling of electrical system

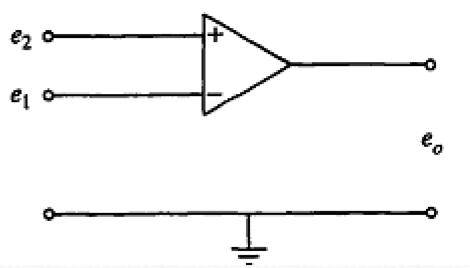
- Analogous to Kirchhoff's laws for networks is d'Alembert's law for mechanical systems which is stated as follows: D'Alembert's law of forces: The sum of all forces acting upon a point mass is equal to zero.

Element	Physical variable	Linear operator	Inverse operator
Electrical Networks			
Resistor R	Voltage v(t) Current i(t)	$v(t) = Ri(t)$	$i(t) = \frac{1}{R}v(t)$
Inductor L		$v(t) = L\frac{d}{dt}i(t)$	$i(t) = \frac{1}{L}\int_0^t v(t)\,dt$
Capacitor C		$v(t) = \frac{1}{C}\int_0^t i(t)\,dt$	$i(t) = C\frac{d}{dt}v(t)$
Mechanical Systems			
Friction coefficient B	Force f(t) Velocity v(t)	$f(t) = Bv(t)$	$v(t) = \frac{1}{B}f(t)$
Mass m		$f(t) = m\frac{d}{dt}v(t)$	$v(t) = \frac{1}{m}\int_0^t f(t)\,dt$
Spring constant K		$f(t) = K\int_0^t v(t)\,dt$	$v(t) = \frac{1}{K}\frac{d}{dt}f(t)$

Mathematical Modeling Of Electronic Circuits

- *Operational amplifiers,* often called *op-amps,* are important building blocks in modem electronic systems. They are used in filters in control systems and to amplify signals in sensor circuits.

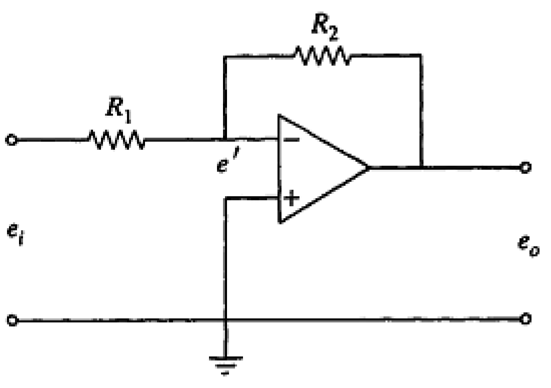
$$e_o = K(e_2 - e_1) = -K(e_1 - e_2)$$



- The input e1 to the minus terminal of the amplifier is inverted; the input e2 to the plus terminal is not inverted.)

Mathematical Modeling Of Electronic Circuits

- Let us obtain the voltage ratio *eo/ei*. In the derivation, we assume the voltage at the minus terminal as *e'*. This is called an *imaginary short*. Consider again the amplifier system



$$i_1 = \frac{e_i - e'}{R_1}, \qquad i_2 = \frac{e' - e_o}{R_2}$$

$$\frac{e_i - e'}{R_1} = \frac{e' - e_o}{R_2}$$

- $e' = 0$. Hence, we have

$$\frac{e_i}{R_1} = \frac{-e_o}{R_2}$$

$$e_o = -\frac{R_2}{R_1}e_i$$

Mathematical Modeling Of Electronic Circuits

- Obtain the relationship between the output *eo* and the inputs e1, *e2,* and *e3*

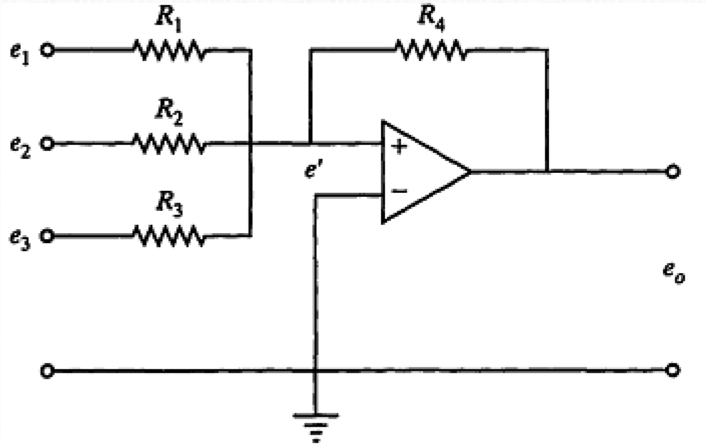
$$i_1 = \frac{e_1 - e'}{R_1}, \qquad i_2 = \frac{e_2 - e'}{R_2}, \qquad i_3 = \frac{e_3 - e'}{R_3}, \qquad i_4 = \frac{e' - e_o}{R_4}$$

$$\frac{e_1 - e'}{R_1} + \frac{e_2 - e'}{R_2} + \frac{e_3 - e'}{R_3} + \frac{e_o - e'}{R_4} = 0$$

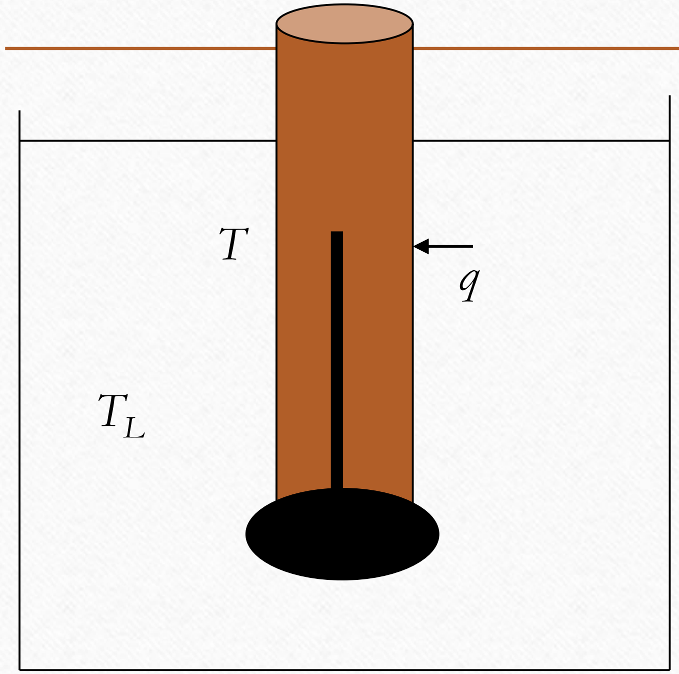
- $e' = 0$. Hence, we have

$$\frac{e_1}{R_1} + \frac{e_2}{R_2} + \frac{e_3}{R_3} + \frac{e_o}{R_4} = 0$$

$$e_o = -\frac{R_4}{R_1}e_1 - \frac{R_4}{R_2}e_2 - \frac{R_4}{R_3}e_3$$



Modeling of Thermal System



q is the net rate of heat flow

C is the capacitance

R is the thermal resistance

$$q = \frac{T_L - T}{R}$$

$$q_1 - q_2 = C\frac{dT}{dt}; q = C\frac{dT}{dt}$$

$$C\frac{dT}{dt} = \frac{T_L - T}{R}$$

$$RC\frac{dT}{dt} + T = T_L$$

Mathematical Modeling Of A Thermal System

- Input – output = stored

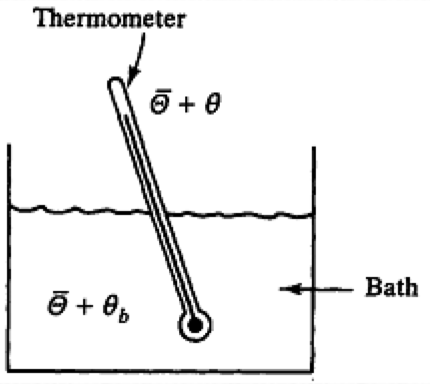
$$q_i(t) - q_o(t) = q_{stored}$$

$$q_{stored} = c\,\frac{d\theta(t)}{dt}$$

$q_o = K\Delta\theta$

- The coefficient K is given by

$$K = \frac{kA}{\Delta X} \quad \text{for conduction}$$
$$= HA \quad \text{for convection}$$

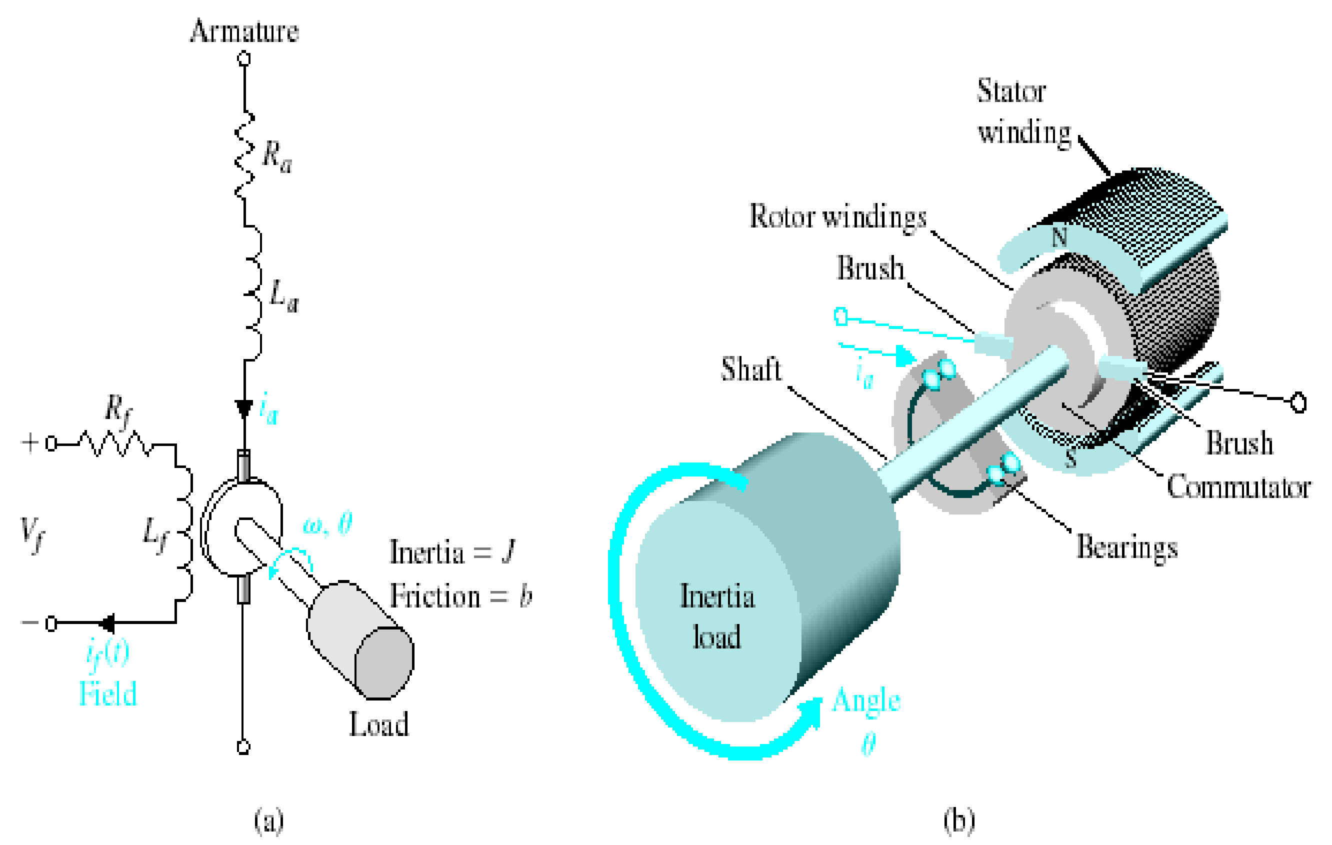


$$R = \frac{\text{change in temperature difference}}{\text{change in heat flow rate}}$$

$$R = \frac{d(\Delta\theta)}{dq} = \frac{\Delta\theta}{q}$$

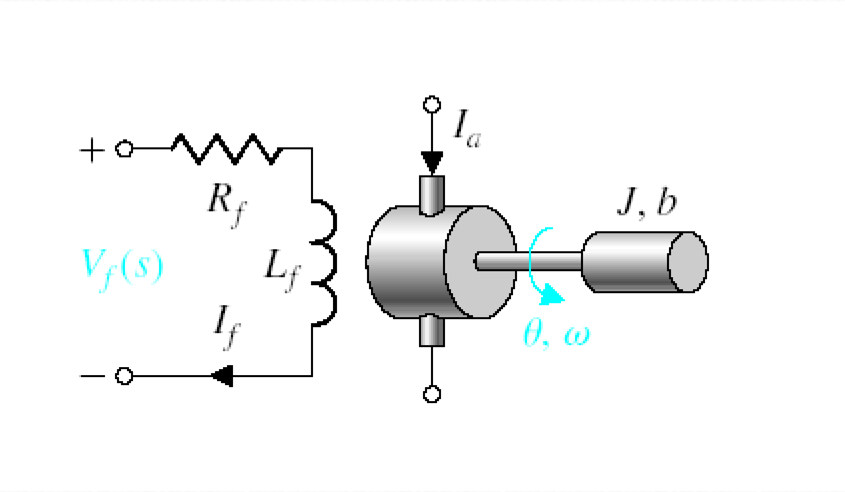
$$RC\frac{d\theta}{dt} + \theta = \theta_b$$

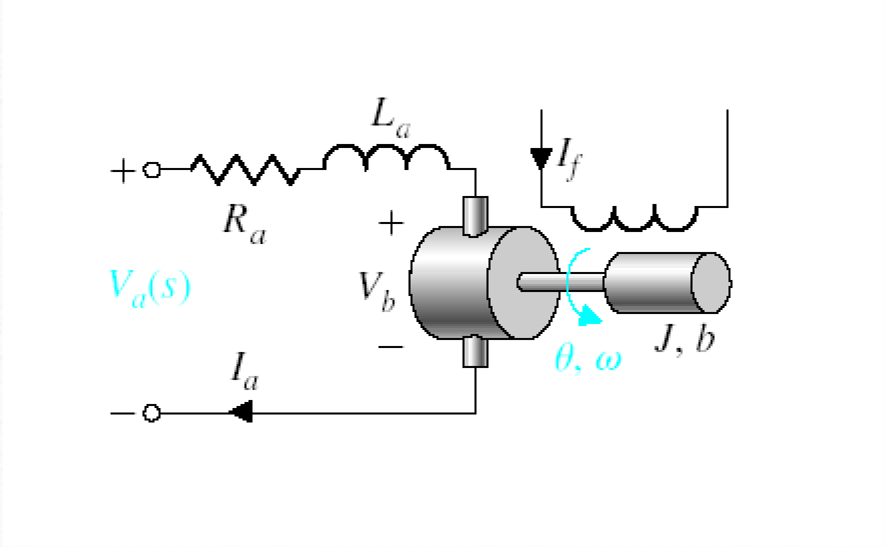
Modeling of Motors



A dc motor (a) wiring diagram and (b) sketch.

Modeling of Motors





$$\frac{\theta(s)}{V_f(s)} = \frac{K_m}{s\cdot(J\cdot s + b)\left(L_f\cdot s + R_f\right)}$$

$$\frac{\theta(s)}{V_a(s)} = \frac{K_m}{s\cdot\left[\left(R_a + L_a\cdot s\right)(J\cdot s + b) + K_b\cdot K_m\right]}$$

Analogy

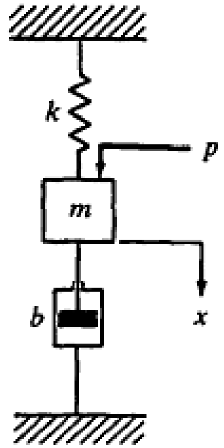
- **Force – voltage analogy**

$$L\frac{di}{dt} + Ri + \frac{1}{C}\int i\,dt = e$$

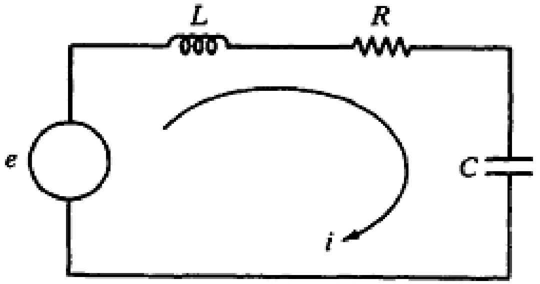
In terms of the electric charge q, this last equation becomes

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q = e$$

Mechanical Systems	Electrical Systems
Force p (torque T)	Voltage e
Mass m (moment of inertia J)	Inductance L
Viscous-friction coefficient b	Resistance R
Spring constant k	Reciprocal of capacitance, $1/C$
Displacement x (angular displacement θ)	Charge q
Velocity $\dot{x}$ (angular velocity $\dot{\theta}$)	Current i



$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = p$$



$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q = e$$

Analogy

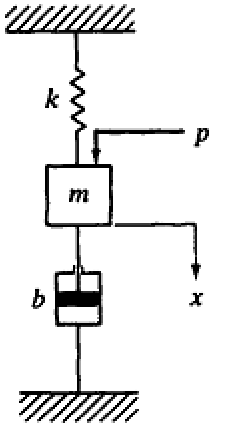
- **Force – current analogy**

$$\frac{1}{L}\int e\,dt + \frac{e}{R} + C\frac{de}{dt} = i_s$$

In terms of the magnetic flux ψ, this last equation becomes

$$C\frac{d^2\psi}{dt^2} + \frac{1}{R}\frac{d\psi}{dt} + \frac{1}{L}\psi = i_s$$

Mechanical Systems	Electrical Systems
Force p (torque T)	Current i
Mass m (moment of inertia J)	Capacitance C
Viscous-friction coefficient b	Reciprocal of resistance, $1/R$
Spring constant k	Reciprocal of inductance, $1/L$
Displacement x (angular displacement θ)	Magnetic flux linkage ψ
Velocity $\dot{x}$ (angular velocity $\dot{\theta}$)	Voltage e



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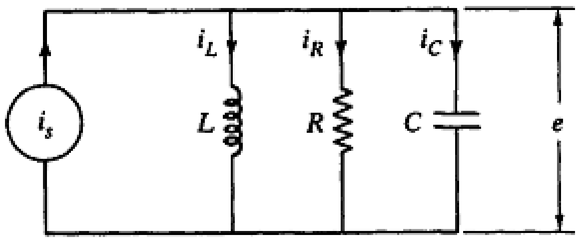
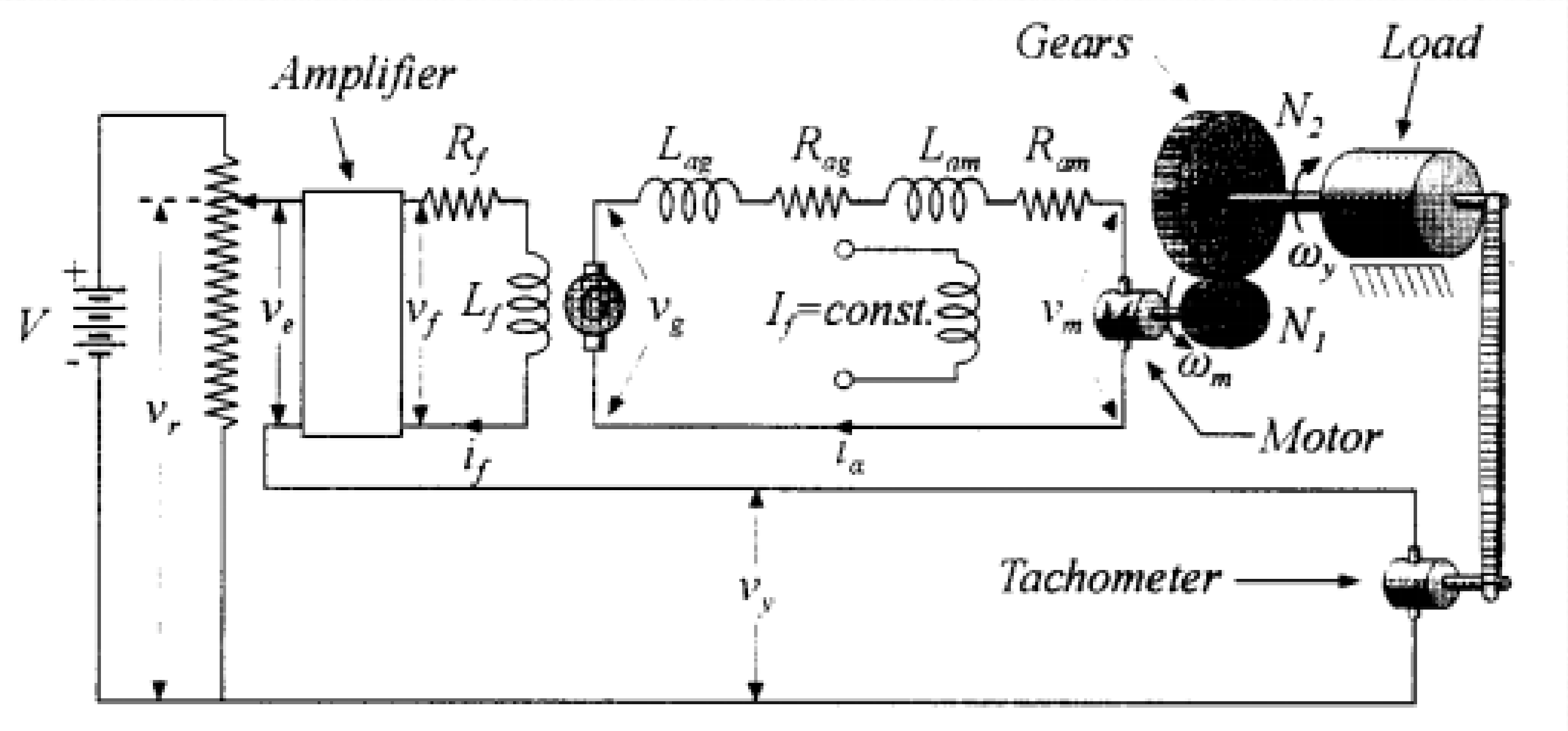
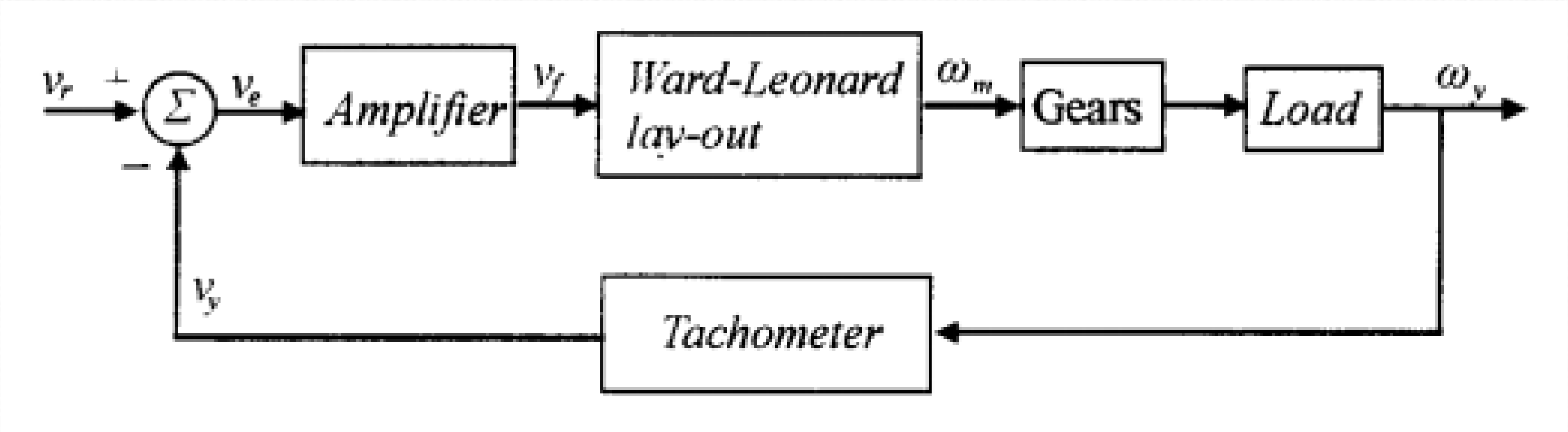


$$C\frac{d^2\psi}{dt^2} + \frac{1}{R}\frac{d\psi}{dt} + \frac{1}{L}\psi = i_s$$

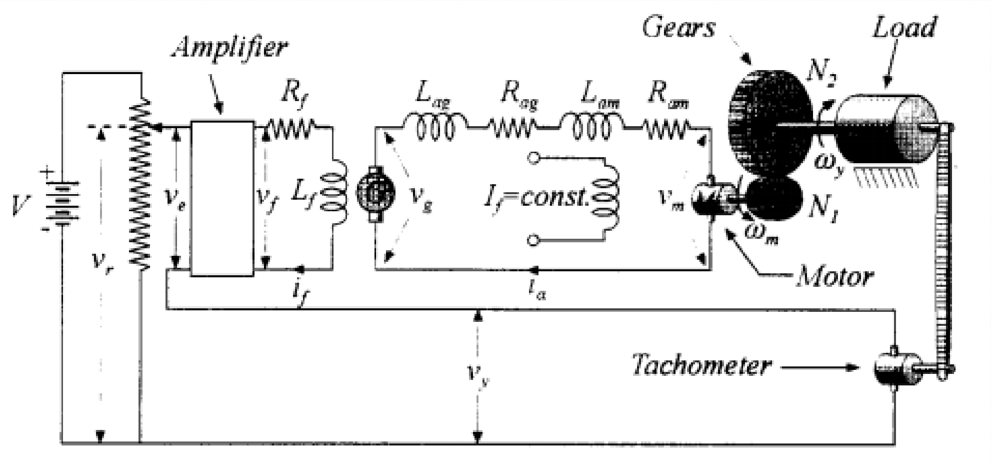
Modeling of Motors

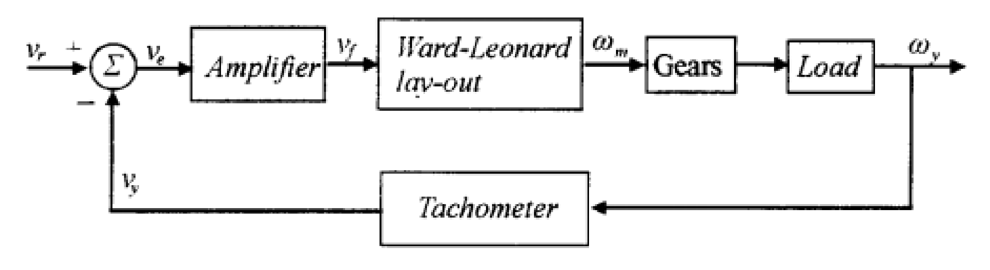


Modeling of Motors



Modeling of Motors





Mathematical Modeling

The equations of the Ward–Leonard layout are as follows . The Kirchhoff's law of voltages of the excitation field of the generator G is

$$v_f = R_f i_f + L_f \frac{di_f}{dt}$$

The voltage v_g of the generator G is proportional to the current i_f, i.e.,

$$v_g = K_g i_f$$

The voltage v_m of the motor M is proportional to the angular velocity ω_m, i.e.,

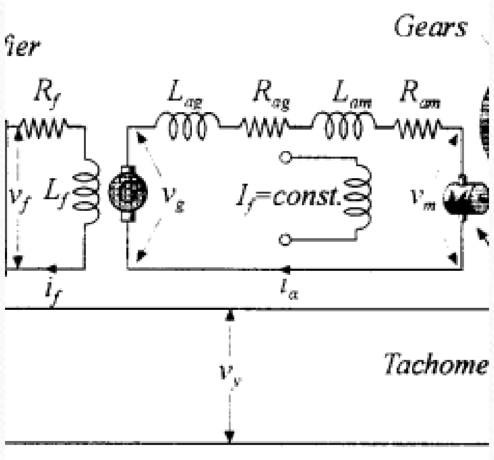
$$v_m = K_b \omega_m$$

The differential equation for the current i_a is

$$R_a i_a + L_a \frac{di_a}{dt} = v_g - v_m = K_g i_f - K_b \omega_m$$

The torque T_m of the motor is proportional to the current i_a

$$T_m = K_m i_a$$

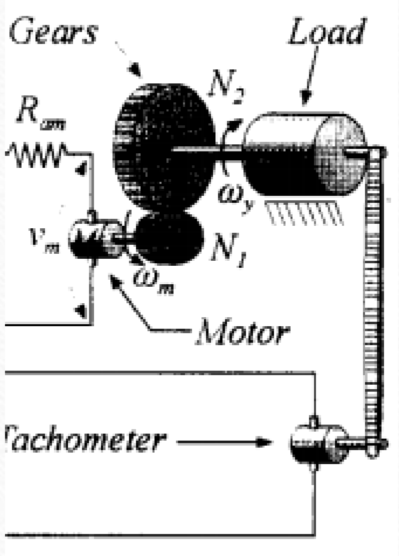


Mathematical Modeling

The equations of the Ward–Leonard layout are as follows . The Kirchhoff's law of voltages of the excitation field of the generator G is

The rotational motion of the rotor is described by

$$J_m^* \frac{d\omega_m}{dt} + B_m^* \omega_m = K_m i_a$$



where $J_m^* = J_m + N^2 J_L$ and $B_m^* = B_m + N^2 B_L$, where $N = N_1/N_2$. Here, J_m is the moment of inertia and B_m the viscosity coefficient of the motor: likewise, for J_L and B_L of the load.

where use was made of the relation

$$\omega_y = N\omega_m.$$

The tachometer equation

$$v_y = K_t \omega_y$$

the amplifier equation

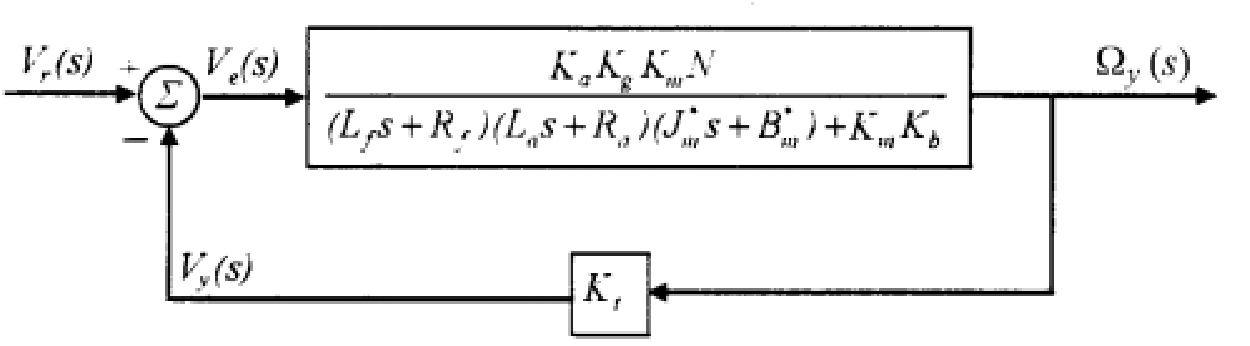
$$v_f = K_a v_e$$

Mathematical Modeling

The mathematical model of the Ward–Leonard layout are as follows .

$$\frac{\Omega_y(s)}{V_f(s)} = \frac{K_g K_m N}{(L_f s + R_f)[(L_a s + R_a)(J_m^* s + B_m^*) + K_m K_b]}$$

$$\frac{\Omega_y(s)}{v_e(s)} = \frac{K_a K_g K_m N}{(L_f s + R_f)[(L_a s + R_a)(J_m^* s + B_m^*) + K_m K_b]}$$



Analogy

- **Force – voltage analogy**

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In terms of the electric charge q, this last equation becomes

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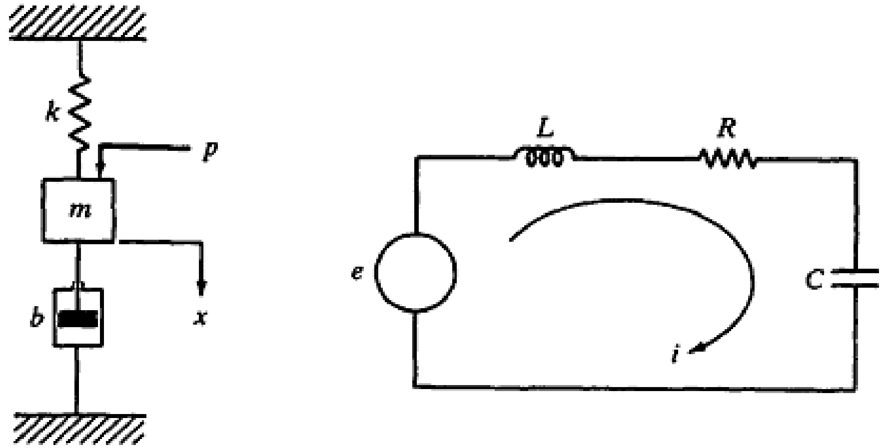

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = p$$

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q = e$$

Analogy

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In terms of the magnetic flux *ψ*, this last equation becomes

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Mechanical Systems	Electrical Systems
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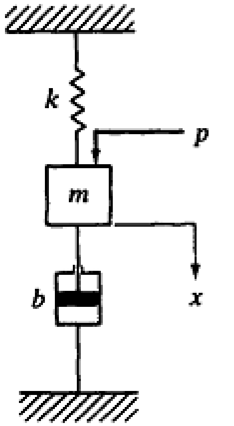
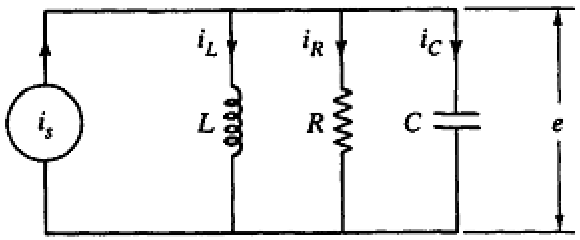
Velocity $\dot{x}$ (angular velocity $\dot{\theta}$)	Voltage e



$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = p$$



$$C\frac{d^2\psi}{dt^2} + \frac{1}{R}\frac{d\psi}{dt} + \frac{1}{L}\psi = i_s$$

Model Examples

- Stepper motor